Ph.D. DIAGNOSTIC EXAMINATION
Example One

Modeling, Simulation and Visualization Engineering (MSVE) Department
Modeling and Simulation Graduate Program
Batten College of Engineering and Technology
Old Dominion University

This examination contains seven problems; you are to complete six of the seven problems. The problem that you choose to omit must be indicated clearly on the front cover sheet. The format for this examination is closed-book, closed-notes and the use of a calculator is not permitted. The time allotted for this examination is three hours.

<table>
<thead>
<tr>
<th>Problem</th>
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NAME:___________________________________________
Problem One (20 points) – Computer Science Concepts

Name:_____________________________

Part A: (10 points)
Some computer scientists argue that agent-based modeling is no different than object oriented programming. Contrast similarities and differences between these two approaches and provide your views and reasoning on which side you support.

Part B: (10 points)
In your opinion what computer science concept is most relevant to modeling and simulation? Justify your answer and explain thoroughly the concept you have chosen.
Problem Two (20 points) – Mathematical Concepts

Name:______________________________

Part A: Calculus (6 points)
An important concept of calculus is “continuity”.

1) Define continuity.

2) Give an example of a function F: \( \mathbb{R} \rightarrow \mathbb{R} \) that is continuous everywhere.

3) Give an example of a function G: \( \mathbb{R} \rightarrow \mathbb{R} \) that is discontinuous at a single point.

Part B: Linear Algebra (6 points)
An important concept of linear algebra is “linear transformation”.

1) Complete the following definition of a linear transformation.
   Definition. Let \( \mathbf{A} \) and \( \mathbf{B} \) be vector spaces. A linear transformation \( \mathbf{T} \) from \( \mathbf{A} \) to \( \mathbf{B} \), written \( \mathbf{T}: \mathbf{A} \rightarrow \mathbf{B} \), is a function that assigns to each vector \( \mathbf{a} \in \mathbf{A} \) a vector \( \mathbf{T}(\mathbf{a}) = \mathbf{b} \in \mathbf{B} \) such that the following two properties hold:

   Property 1: ______________________________________________

   Property 2: ______________________________________________

2) Determine if each of the following functions \( \mathbf{T}: \mathbb{R}^3 \rightarrow \mathbb{R}^2 \) are linear transformations. Justify your answers.

   (a) \( \mathbf{T}(x, y, z) = (x + y, 2z) \)

   (b) \( \mathbf{T}(x, y, z) = (x+y+z, 1) \)

Part B: Differential Equations (8 points)
Solve the following differential equation for \( t > 0 \).

\[
\frac{d^2x(t)}{dt^2} + 3\frac{dx(t)}{dt} + 2x(t) = 1; \quad x(0) = 1, \quad \left. \frac{dx(t)}{dt} \right|_{t=0} = 1.
\]
Problem Three (20 points) – Probability and Statistics

Name:_____________________________

Part A: Probability (10 points)
In this problem, we investigate the concept of random variables.

1) Define the following terms.
   a) Random Variable X
   b) Cumulative Distribution Function F(x)
   c) Probability Density Function p(x).

2) The probability density function for a random variable X is given by:

   \[ p(x) = z\delta(t) +zx[u(t) - u(t - 2)] + 2z\delta(t - 2) \]

   where \( \delta(\bullet) \) is the unit impulse function and \( u(\bullet) \) is the unit step function.

   a) Determine the value of the constant \( z \).
   b) Determine the probability that \( X = 1.5 \).
   c) Determine the probability that \( X \leq 1 \).

Part B: Statistics (10 points)
Statistical estimation is the process of estimating a population parameter based upon knowledge of a sample statistic. The sample statistic used in estimating a population parameter is called an estimator. The most popular form of an estimator in M&S is the confidence interval estimate. In this problem, we investigate the confidence interval estimate of the mean.

1. The words “precision” and “reliability” are used when describing a confidence interval estimate. Define the meaning of these two terms and comment on the relationship between the terms.

2. Estimators often are characterized by the following terms: (a) unbiased; (b) consistent; and (c) efficient. Define the meaning of each of these terms.

3. Suppose that \( \{ X_1, X_2, \ldots, X_n \} \) are random variables (observations) from some population and suppose that we are interested in developing a confidence interval estimate of the population mean.
a. Define a sample mean that serves as an unbiased, consistent, and efficient estimator for the population mean.

b. Define a sample variance that serves as an unbiased, consistent, and efficient estimator for the population variance.

c. Explain how these quantities (the sample mean and sample variance) are combined to form a confidence interval estimate of the population mean.
Problem Four (20 points) – Discrete Event Simulation

Name: _______________________________

The spreadsheet shown on the following page represents the first several events in a hand simulation of a simple processing system. Answer the following questions concerning this hand simulation.

Part A: (8 points)
Complete the last two rows shown in the simulation spreadsheet. The following are the “Statistical Accumulators” column definitions for the simulation spreadsheet.

- P, Number produced
- N, Number that have passed through the queue
- ΣD, Total of the times spent in queue
- D*, Max time in queue
- ΣF, Total of flow-times
- F*, Max flow-time
- ∫Q, Area under queue-length curve Q(t)
- Q*, Max of Q(t)
- ∫B, Area under server-busy curve B(t)

Part B: (12 points)
Determine the following performance measures for this system.

1. Determine the AVERAGE NUMBER OF ENTITIES IN QUEUE for the hand simulation.

2. Determine the AVERAGE DELAY IN QUEUE for the hand simulation.

3. Determine the AVERAGE FLOW-TIME for the hand simulation.

4. Determine the PERCENTAGE UTILIZATION OF THE SERVER for the hand simulation.
<table>
<thead>
<tr>
<th>Just-Finished Event</th>
<th>Variables</th>
<th>Attributes</th>
<th>Statistical Accumulators</th>
<th>Event Calendar</th>
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<tbody>
<tr>
<td>Entity No.</td>
<td>Time</td>
<td>Type</td>
<td>Q(t) B(t)</td>
<td>Arrival Times: (In Queue)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>START</td>
<td>0</td>
<td>0</td>
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<tr>
<td>8</td>
<td>21</td>
<td>ARR</td>
<td>3</td>
<td>22</td>
</tr>
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</table>
Problem Five (20 points) – System Modeling Concepts

Name:_______________________________

Part A: Bayesian Inference (10 points)
Given the following table of probabilities, use Bayesian inferences to determine the Chance of a Clear Sky Day given it is Raining.

<table>
<thead>
<tr>
<th>If there is . . .</th>
<th>Rain</th>
<th>No Rain</th>
</tr>
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<tbody>
<tr>
<td>Clear Sky (.25)</td>
<td>.05</td>
<td>.95</td>
</tr>
<tr>
<td>Moderate Sky (.50)</td>
<td>.30</td>
<td>.70</td>
</tr>
<tr>
<td>Stormy Sky (.25)</td>
<td>.80</td>
<td>.20</td>
</tr>
</tbody>
</table>

\[
\text{Prob}(P|D) = \frac{\text{Prob}(D|P) \cdot \text{Prob}(P)}{\text{Prob}(D)}
\]

\[
\text{Prob}(P_x|D) = \frac{\text{Prob}(D|P_x) \cdot \text{Prob}(P_x)}{\sum \text{Prob}(D|P_i) \cdot \text{Prob}(P_i)}
\]
Problem Five Continued

Part B: Systems Dynamics (10 points)
Reference the systems dynamics model below.

![Diagram of systems dynamics model]

a. Derive the appropriate equations?

b. Provide a functional block model that implements the derived equations.
Problem Six (20 points) – Analysis Concepts

Name: ________________________________

Part A: Minimal Spanning Tree (6 pts)
Find the minimal spanning tree of the following network:

Part B: Simplex Method (14 Points)
Utilize the following problem to answer the questions:

Max \( 12x_1 + 18x_2 + 10x_3 \)

s.t. \( \begin{align*} 2x_1 + 3x_2 + 4x_3 & \leq 50 \\
       x_1 - x_2 - x_3 & \geq 0 \\
       x_2 - 1.5x_3 & \geq 0 \end{align*} \)

\( x_1, x_2, x_3 \geq 0 \)

Questions:

a) Solve the problem using the simplex method. What is the optimal solution? What is the optimal value of the objective function? (12 pts).

b) Are there alternate optimal solutions? Why or why not? (2 pts).
Problem Seven (20 points) – Visualization Concepts

Name:_______________________________

Part A: (6 points)
Check if the following operations are valid or not, where upper-case letters represent points and lower-case letters represent vectors.

1. 1.8v + 3.5w – 1.3u
2. 10v + 2.8P + 0.1Q – 0.9R
3. −28P + 27Q + 13v

Part B: (6 points)
1. Represent a point \( P \) \((233, -189, 600)\) and a vector \( v \) \((-103, -169, 423)\) in homogeneous coordinates.

2. A point is translated by a displacement vector \((68, -101, 23)\). Represent this translation by a 4 x 4 matrix using homogeneous coordinates. Also calculate its inverse transformation.

3. Describe the coordinate systems in the OpenGL graphics pipeline.

Part C: (8 points)
A camera is located at the location \((0,3,0)\) in the world coordinate system, pointing to the origin \((0,0,0)\). Assume that the up-direction of the camera is \((0,0,1)\) and the positive \(z\)-axis of camera coordinate system points from the center of camera to the back of the camera. Derive the transformation matrix that transforms world coordinates to camera coordinates.