Ph.D. DIAGNOSTIC EXAMINATION
EXAMPLE 3

Modeling, Simulation and Visualization Engineering (MSVE) Department
Modeling and Simulation Graduate Program
Batten College of Engineering and Technology
Old Dominion University

This examination contains seven problems; you are to complete six of the seven problems. The problem that you choose to omit must be indicated clearly on the front cover sheet. The format for this examination is closed-book, closed-notes and the use of a calculator is not permitted. The time allotted for this examination is three hours.

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NAME:___________________________________________
Problem One (20 points) – Computer Science Concepts

Name:_____________________________

Define Object-Oriented. What are its characteristics? How does it differ from non-object-oriented approaches? What are the major parts of an object model? Describe these parts and provide examples.
Problem Two (20 points) – Mathematical Concepts

Name:______________________________

Part A: (7 points)
Calculus. An important calculus operation is the differentiation of an integral.

1) Show how to calculate the indicated derivative where \( a(t) \) and \( b(t) \) are functions of \( t \) and \( f(t, \tau) \) is a function of both \( t \) and \( \tau \).

\[
\frac{d}{dt} \left[ \int_{a(t)}^{b(t)} f(t, \tau) \, d\tau \right] =
\]

2) Let \( f(t) = \sin(t) \cos(t) \). Use your answer to (1) above to compute \( g(t) \) where

\[
g(t) = \frac{d}{dt} \left[ \int_{0}^{\infty} f(\tau) \, d\tau \right].
\]

Part B: (6 points)
Linear Algebra. An important concept of linear algebra is the “inverse function”.

1) Let \( f \) be a function mapping from the real numbers (\( \mathbb{R} \)) to the real numbers (\( \mathbb{R} \)); that is \( f : \mathbb{R} \rightarrow \mathbb{R} \). Define \( f^{-1} \) to be the “inverse” for the function \( f \). Define \( f^{-1} \) and state the conditions on \( f \) for the existence of \( f^{-1} \).

2) Determine if each of the following functions \( f : \mathbb{R} \rightarrow \mathbb{R} \) have an inverse. If so, determine the inverse function. If not, explain why.

(a) \( f(x) = mx + b \), where \( m \) and \( b \) are real constants different from zero.

(b) \( f(x) = cx^2 \), where \( c \) is a real constant different from zero.

Part C: (7 points)
Differential Equations. Solve the following differential equation for \( t > 0 \).

\[
\frac{dx(t)}{dt} + 3x(t) = 9t; \quad x(0) = 9.
\]
Problem Three (20 points) – Probability and Statistics

Name: ________________________________

Part A: Pseudo-random numbers (10 points)
Generate the first four pseudo-random 2-digit numbers using the Von Neumann’s mid-squared method with a seed of ‘33’. The method involves iteratively squaring the current number to generate a 4-digit number and then taking the middle two digits of this number as the next pseudo-random number.

1. What can you say about the next 100 generated pseudo random numbers from this seed?

2. Name some of the possible problems with the Von Neumann’s mid-squared method.

3. How would you adapt this approach to generate a random number between 0 and 1? What issues are there with this adaptation?

Part B: Probability (10 points)
The final challenge of a game show is as follows:
- Behind one of three doors is the prize
- The contestant chooses ONE of the doors
- The host opens one of the remaining doors to reveal that it does not have the prize behind it.
- The contestant is now given the choice to stick with the door they chose or to choose the remaining un-opened door. The contestant gets what is behind that door.

Should the contestant stick with their original choice or choose the remaining unopened door? Determine the probability of your choice and show all workings. You can assume, without loss of generality, that the contestant initially choose door one.
Problem Four (20 points) – Discrete Event Simulation

Name: _________________________________

A two-runway airport, which features one runway for take-offs and one runway for landings, is being designed for propeller-driven aircraft. The time to land an airplane is known to be exponentially distributed, with a mean of 2 minutes. If airplane arrivals are assumed to occur at random, what arrival rate can be tolerated if the average wait in the sky is not to exceed 4 minutes?

Formulas:

**M/G/1 queues**

\[ \rho = \frac{\lambda}{\mu}, \quad P_0 = 1 - \rho \]

\[ L = \rho + \frac{\rho^2 (1 + \alpha^2 \mu^2)}{2(1 - \rho)}, \quad L_Q = \frac{\rho^2 (1 + \alpha^2 \mu^2)}{2(1 - \rho)} \]

\[ w = \frac{1}{\mu} + \frac{\lambda(1/\mu^2 + \sigma^2)}{2(1 - \rho)}, \quad w_Q = \frac{\lambda(1/\mu^2 + \sigma^2)}{2(1 - \rho)} \]

**M/M/1 queues**

\[ \rho = \frac{\lambda}{\mu}, \quad P_n = (1 - \rho)^n \]

\[ L = \frac{\lambda}{\mu - \lambda} = \frac{\rho}{1 - \rho}, \quad L_Q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{\rho^2}{\mu(1 - \rho)} \]

\[ w = \frac{1}{\mu - \lambda} = \frac{1}{\mu(1 - \rho)}, \quad w_Q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{\rho}{\mu(1 - \rho)} \]

**M/M/c queues**

\[ \rho = \frac{\lambda}{c \mu} \]

\[ P_0 = \left[ \sum_{n=0}^{c-1} \frac{(\lambda/\mu)^n}{n!} \right] + \left[ \frac{\lambda}{\mu} \left( \frac{1}{c!} \left( \frac{c \mu}{\lambda - \mu} \right) \right) \right]^{-1} \]

\[ L = \frac{c \rho}{c(c!)^2(1 - \rho)^2} = c \rho + \frac{\rho P(L(\infty) \geq c)}{1 - \rho} \]

\[ w = \frac{L}{\lambda}, \quad w_Q = w - \frac{1}{\mu} \]
Problem Five (20 points) – System Modeling Concepts

Name: ________________________________

BAYESIAN BELIEF NETWORKS
Mr. Holmes leaves his house. The grass is wet (W=True) in front of his house. Two reasons are possible. Either it rained (R=True) or the sprinkler (S=True) has been on during the night or both. Then, Mr. Holmes looks at the sky and finds it is cloudy (C=True). Since when it is cloudy, usually the sprinkler is off (S=False) and it is more probable that it rained, he concludes it is more likely that rain caused the grass to be wet.

1. Using the Chain Rule, expand the following: (4 pts)
   \[ P(C, S, R, W) = \]

2. How can the resulting terms from #1 be simplified? Show the result. (4 pts)
   \[ P(C, S, R, W) = \]

3. Given the grass is wet, calculate the probability that the sprinkler was on? (4 pts)

4. Given the grass is wet and it rained, calculate the probability that the sprinkler was on? (4 pts)

5. Describe the concept of “explaining away.” (4 pts)
Problem Six (20 points) – Analysis Concepts

Name:______________________________

Part A: (8 points)
Suppose that we have I.I.D. data $X_1, X_2, \ldots, X_n$, with known standard deviation $\sigma$. We know that an approximate $100(1-\alpha)\%$ confidence interval for the mean of $X_1$ is given by:

$$\bar{X}_n \pm \frac{z_{\alpha/2} \sigma}{\sqrt{n}}$$

a. Explain why this confidence interval is only approximate, in general. (4 points)

b. Give a situation where the word “approximate” is not needed. That is, provide an example when the above confidence interval is exact at the $100(1-\alpha)\%$ confidence level. (4 points)

Part B: (6 points total, 2 each)
Are the following statements true or false? Justify your answer in each case.

a. The batch means method is suitable for analyzing both terminating simulation and steady state simulation.

b. To deal with initialization bias, it is critical to have a warm-up period when performing a terminating simulation.

c. In a terminating simulation, the system output typically depends on the initial system state.

Part C: (6 points)
In a steady–state simulation, suppose that we have used the truncated replications method to construct a confidence interval for our performance measure. However, we are not satisfied with the width of the confidence interval. Provide two ways to reduce the width of the confidence interval. Which one is preferred? Why?
Problem Seven (20 points) – Visualization Concepts

Name:_______________________________

Part A: (6 points)
Homogeneous coordinates are used in computer graphics and visualization to represent 3D entities, such as points and vectors.

1. Explain why homogeneous coordinates are used to represent points and vectors.

2. There are two 3D entities: \( a = [4 \ 2 \ 3 \ 0]^T \), and \( b = [3 \ 20 \ 1 \ 1]^T \). Determine whether \( a \) and \( b \) are a point or a vector.

3. What do the last components in \( a = [1 \ 2 \ 3 \ 1]^T \) and \( b = [3 \ 2 \ 1 \ 0]^T \) represent?

Part B: (6 points)

1. Describe and explain different types of matrices used by OpenGL. What is a matrix stack and how is it used?

2. What is a convex object? Why does computer graphics prefer to use convex polygons such as triangles?

3. Describe the differences among local (or object) coordinates, world coordinates, and camera coordinates.

Part C: (8 points)

1. Assuming that \( P_1, P_2, \ldots, P_n \) are points and \( \alpha_1, \alpha_2, \ldots, \alpha_n \) are real numbers, prove that the result of the following computation:
   \[
P = \alpha_1 P_1 + \alpha_2 P_2 + \cdots + \alpha_n P_n
   \]
   is a point if \( \alpha_1 + \alpha_2 + \cdots + \alpha_n = 1 \) and is a vector if \( \alpha_1 + \alpha_2 + \cdots + \alpha_n = 0 \).

2. Compute the normal of the plane represented by \( 2x + 4y + 5z + 4 = 0 \).